PROBLEM SOLVING

– parts 2 and 3

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Finding rules and describing patterns Logic problems

This guidance is part of a series of materials from the Primary National Strategy designed to help all staff involved in the teaching of mathematics to:

- identify problems of a particular type and the strategies that children can use to solve them;
- construct teaching sequences for teaching problem solving;
- incorporate problem solving within the mathematics curriculum;
- develop children's reasoning and explanation skills;
- engage in whole-staff discussions on the above.

These materials are designed to follow on from the first CPD pack (DfES 0247-2004 G) which focused on 'Finding all possibilities'. It is recommended that those materials are used before these.

The materials together aim to support schools to:

- identify the principles of good practice when teaching problem solving;
- identify the progression in children's strategies that should inform learning and expectations.

The first CPD pack should have prompted schools to begin deciding on their policy for improving the teaching and learning of problem solving. These materials are designed to help them to build on those decisions through a focus on two other areas:

- finding rules and describing patterns;
- logic problems.

Proposed model for use of these materials in school

This set of materials suggests four staff meetings, perhaps two in one term and two in a following term, to support the lessons it contains:

- Two one-hour meetings focus on finding rules and describing patterns, and discuss progression in expectations and assessment around problem solving.
- A set of lessons focuses on finding rules and describing patterns. Teachers are asked to use them with their own classes between the first and second staff meetings.
- Two further meetings focus on logic problems.
- A set of lessons focuses on logic problems. These are again used between the two staff meetings.

Module 2: Finding rules and describing patterns

Objectives

- To build on the work from the previous module by broadening the understanding of problem solving
- To consider whole-school progression within problem solving

Notes to guide whole-staff discussion: meeting 1

60 minutes

Outline of staff meeting

1. Introduction; building on 'Finding all possibilities'
2. Progression in 'Finding rules and describing patterns', Years 1–6
3. Preparing the lessons for teaching
4. Where next?

Objectives

- To build on the work from the previous module by broadening the understanding of problem solving
- To identify the skills and strategies to be developed and demonstrated in a lesson that will inform the next meeting

Key messages

- Children need to develop their pattern skills.
- Children can be given the same problem at different stages through the primary school, provided that the teacher has considered the expectations appropriate to the year.
- Children need to be given opportunities to practise their explanations.
- Generalisation is an important aspect of mathematics which children need to develop from describing, predicting and explaining.

Resources

- Copies of the school policy for problem solving, or notes on the principles agreed
- Copies of the lessons in the 'Finding rules and describing patterns' pack

Introduction (5 minutes)

Explain that we are continuing to look at the skills and strategies needed to help children become proficient problem solvers. Remind teachers that the Primary National Strategy has classified five different types of problem as follows:

- Finding all possibilities;
- Logic problems;
- Finding rules and describing patterns;
- Diagram problems and visual puzzles;
- Word problems.

Remind participants that they have previously focused on the types of problems that involve finding all possibilities, and that they taught two lessons between staff meetings and then discussed:

- the features of good problem-solving lessons;
- the principles of good practice when teaching problem solving;
- how they would help children develop some of the important strategies and skills needed in problem solving in the school.

In this meeting we are going to look at problems that involve finding rules and describing patterns.

Preparing to teach the lessons (20 minutes)

Explain that, as before, the teachers will be asked to teach some lessons with their classes between this and the next staff meeting so that they can discuss what problem-solving strategies children have demonstrated and what strategies need to be developed.

There are two different strands to these lessons. In the first strand teachers consider the skills children need in order to solve problems involving patterns, and in the second strand teachers look at the application of these skills.

For the second strand the lessons teachers are asked to do with their classes take the same problem from Years 1–2 through Years 3–4 to Years 5–6, but at each stage they require more from the children.

Say that you are now going to look at these lessons together in order to get an overview of the progression of skills and strategies that are developed and the expectations for each stage.

Explain that you will give only a brief outline of the Years 1–2 and Years 3–4 lessons and will look in more detail at the Years 5–6 lesson. Then everyone will have time to look at their lessons in detail.

Years 1–2 lesson: Teddy's birthday candles

Ask all participants to turn to the lesson and give a brief (3–4 minute) outline of it. This might draw attention to:

- the problem;
- the lesson objectives;
- the focus vocabulary;
- the necessary prior knowledge;

- the layout of the lesson plans. As before, the lesson is set out with the main 'story' of the lesson in the middle column; the left-hand column suggests how the lesson might be modified; and the right-hand column consists of illustrative thoughts behind the planning that help to focus on key issues such as children giving explanations;
- the patterns children might find, including those suggested by the way they record;
- the range of numbers children are expected to work with;
- the summary of strategies that have been used.

Years 3–4 lesson: Birthday candles

Now turn to the lesson for Years 3–4 and give a brief (3–4 minute) outline of it. Ask teachers to draw out the differences expected from the Year 3 and Year 4 children.

The outline might draw attention to:

- the objectives: at this stage children are asked to continue number sequences and explain the generalisations they make;
- the need for children to create a rule to find the number of candles someone of any age had blown out in total on their birthday;
- the extension, which challenges children to calculate the total number of candles blown out on any birthday using the strategy for adding several numbers.

Years 5–6 lesson: Mrs Gillespie's 73rd birthday (10 minutes)

Tell teachers that they are going to try the Years 5–6 problem themselves.

Explain that Mrs. Gillespie is 73 today and the problem is to work out how many candles she has blown out in her lifetime.

Say that you would like the teachers to work in pairs to decide what kind of record may be useful to work out the problem, reminding them that their recording needs to be systematic and easy to refer to. Then you would like them to work out the answer.

Give teachers a few minutes for this.

It is likely that most teachers will write down all the numbers from 1 to 73 and then add them up. Point out, after a couple of minutes, that this will take a long time and that you are going to try and move to a more efficient strategy.

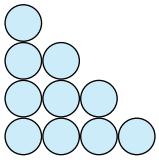
It may be necessary to go through the following explanation to help teachers to understand the algebraic solution to the problem.

Draw attention to the lesson objective, including 'by the end of the lesson the children will be able to express the general term in words and begin to do this algebraically.'

Say you are going to simplify the problem (a problem-solving strategy) and use a smaller number.

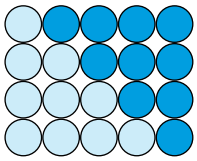
Draw the first four rows of the pattern on the board, saying that you are going to look at the pattern when Mrs Gilllespie was four years old. Ask teachers to consider:

- the way the diagram grows;
- the number of candles at the side of the diagram and at the bottom of the diagram;
- the numbers generated;
- the shape it makes.



Ask the teachers to describe what they notice. Draw out the following.

- The number of candles at the side and at the bottom of the shape is the same: 4.
- This number is the same as Mrs Gillespie's age.
- The number of candles in total each year rises by Mrs Gillespie's age: at age 4 there are 4 more candles (10 in total) than when she was 3 (6 in total). Remind the teachers that numbers in a sequence like this are called triangular numbers, for obvious reasons.
- The shape formed by the candles is a right-angled triangle.
- It is possible to make a rectangle out of this arrangement by copying the triangle, inverting it, rotating it, and placing it on top.



Describe this rectangle. This has dimensions 4 (Mrs Gillespie's age) and 5 (Mrs Gillespie's age plus 1).

What is the relationship between the rectangle and the number of candles? (Half the rectangle is equal to the number of candles, as we created the rectangle by using two of the triangles.)

The number of candles in the rectangle is $4 \times 5 = 20$.

Therefore the number of candles Mrs Gillespie has blown out by her fourth birthday is $(4 \times 5) \div 2 = 10$.

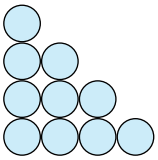
Ask what the rectangle would look like if Mrs Gillespie were 10. (10 by 11.)

How could you calculate the number of candles Mrs Gillespie has blown out by her tenth birthday? ($10 \times 11 \div 2 = 55$.)

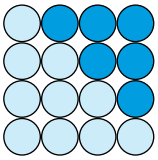
If Mrs Gillespie is N years old what are the dimensions of the rectangle? (N by $N + 1$.)

Therefore the number of candles blown out on Nth birthday is $N \times (N + 1) \div 2$.

- An alternative way of calculating is:

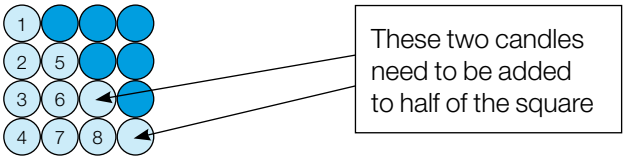


It is possible to make a square out of this arrangement of candles by adding extra candles above the diagonal line: 3 to the first row, 2 to the second row and 1 to the third row. (You might want to point out that necessary prior knowledge includes 'know the relationship between the areas of right-angled isosceles triangles and squares.')



The number of candles in the right-angled triangle is just a bit more than half the number in the square (half the number in the square is 8, while the number in the triangle is 10).

This extra bit comes from the number of candles in the entire diagonal of the square, not just half of it.



To find the number of candles you halve the square and then add on half of the diagonal.

The diagonal equals 4, that is, Mrs Gillespie's age, so half the diagonal is half Mrs Gillespie's age.

To calculate the number of candles:
- Find the square, that is, Mrs Gillespie's age × Mrs Gillespie's age ($4 \times 4 = 16$)
- Halve the square ($16 \div 2 = 8$)
- Add on half of Mrs Gillespie's age ($8 + (4 \div 2)$).

You might want to give teachers a few minutes to explore by extending the sequence to look at the numbers at the 5th, 6th and 7th birthdays and then work out the general rule. The general rule should find that the total number of candles is:
- half the number in the 'square' (or the most recent birthday number squared) plus half the number of candles in the diagonal of the square (or the most recent birthday number)

Expressed algebraically, that is $\frac{n^2}{2} + \frac{n}{2}$ or $(n^2 + n) \div 2$.

Explain that using models and images has helped to find the generalisation. This is a good strategy for being able to explain how a pattern is developing.

Ask teachers to return to the original problem and find the number of candles Mrs Gillespie will have blown out in total on her 73rd birthday (2701).

Another strategy that teachers might think of is:

Write the numbers in a line:
$1 + 2 + 3 + 4\ldots + 70 + 71 + 72 + 73$

Underneath write all the numbers in the reverse direction:
$1 + 2 + 3 + 4\ldots + 70 + 71 + 72 + 73$
$73 + 72 + 71 + 70\ldots + 4 + 3 + 2 + 1$

Add these two lines together:
$74 + 74 + 74 + 4\ldots + 74 + 74 + 74 + 74$
This gives 73 lots of 74 (73×74).

This is twice the total needed to solve the problem, so the answer is $(73 \times 74) \div 2$, or $n \times (n + 1) \div 2$.

Lesson preparation (15 minutes)

Tell teachers that you are now going to give them time to look at the lesson for their stage in detail and prepare to teach it. The Reception and Years 1–2, Years 3–4, and Years 5–6 teachers are to work together.

You would like them to consider the principles they agreed at the previous staff meeting for planning and teaching lessons in problem solving. Remind them of these.

If the following do not form part of the principles you might want teachers to consider the following points:

- How will you introduce the problem to your class and ensure that they understand it?
- What resources do you need so that all children can access the problem at their level?
- How can we remove any barriers that might prevent children from being successful?
- How far will you expect children in your year to get with the given problem? (Consider the ability range.) What strategies might you expect them to use? Are there strategies you will want to develop?
- What recording will you expect from children?
- What kinds of explanation will you expect?
- What skills do children need to access the problem? Do they have these skills? When will you teach them?
- How far do the skills lessons support children in accessing the birthday cake problem?

What next?

Agree with the staff when they might teach the prepared lessons with their classes and when the next staff meeting will be to discuss children's progress.

Say that you would like them to bring examples of children's work that show:

- that they have achieved the lesson objectives;
- a variety of strategies used in solving the problem;
- examples of children's explanations (written, or notes taken from oral explanation);
- examples of children's work from the skills lessons.

Notes to guide whole-staff discussion: meeting 2

60 minutes

Outline of staff meeting

1. Introduction; statements on the three strands of problem solving
2. Using the statements to evaluate teachers' own classes
3. Demonstrating progression
4. Where next?

Objectives

- To evaluate a problem-solving checklist as an aid for planning and assessment
- To agree aspects of problem solving to focus on in each year
- To moderate examples of children's problem solving
- To begin a school portfolio with examples of problem-solving work

Resources

- Handout of statements for discussion about problem solving
- Copies of the school policy for problem solving or notes on the principles agreed
- Copies of the lessons in the 'Finding rules and describing patterns' pack
- Examples from teachers of children's work from Reception, Years 1–2, Years 3–4 and Years 5–6 for finding rules and describing patterns
- Examples of children's work that demonstrate 'Finding all possibilities' (if available)

Introduction (5 minutes)

Ask teachers to sit in phase group: Reception, Years 1–2, Years 3–4s and Years 5–6.

Explain that there will be two aspects to this staff meeting. The first is to review the lessons and draw out any issues, and the second is to consider the progression of children through the school. Explain that you are trying to decide whether a list of statements that break down the skills and strategies needed to be successful problem solvers could be helpful when planning and assessing in your school.

Ask the staff to decide:

- whether the lesson objectives were achieved, and what helped or hindered the process;
- whether the skills lessons help children towards more success with the birthday candle problem lesson.

Distribute the handout of strategies and point out that they are under the three headings of Using and Applying Mathematics:

- Making and monitoring decisions to solve problems
- Communicating
- Reasoning.

Each section has been subdivided into statements with common themes.

Point out that there is no year group associated with any of the statements, and that one of the aims of this meeting is to try and agree which statements might be an appropriate focus for each year or stage.

Give teachers a few minutes to look at the handout. Ask how this information might be used in the school.

Possible suggestions are:

- to use the statements as curricular targets;
- to use a set of the statements to identify whether children are making progress through the school;
- to use the statements to create a school portfolio demonstrating progression in problem solving.

It is not expected that teachers will assess all children against all these statements.

Looking at children's work (20 minutes)

Ask teachers to look at the examples of children's work from their own classes (finding rules and describing patterns, plus finding all possibilities, if available) and match them with the statements on the handout.

Their task is to look for good examples of achievement. When they have found examples of work from their own classes you want them to share the examples with their stage colleagues and then with all the staff.

The aim is:

- to reach agreement about what the statement means;
- to agree the examples of work that demonstrate achievement at an appropriate level. Be prepared to explain why it is appropriate for a particular level, for example use of language, symbols, or generalisations.

Ask teachers to label the work they choose as good examples so that you can keep them for a school portfolio.

Demonstrating progression (20 minutes)

Say that on the handout there are statements that are appropriate for Year 1 through to Year 6.

Rearrange staff into groups with representatives across the age ranges. Ask staff to take a group of statements and to write in the last column which statements they think are particularly appropriate to specific ages. It would be helpful to take a group of statements where teachers have an example of a child's work.

Give teachers a few minutes for this task. Circulate and discuss any issues.

From the previous work the teachers have decided which statement the child's work demonstrates. Look at the piece of work and agree the statement that it illustrates. As a group decide what the next steps for this child might be.

Where next? (10 minutes)

Remind teachers that one of the aims of the meeting was to evaluate the checklist as an aid to planning and assessment. You want to know whether it would help them plan which aspects of problem solving they would need to focus on in each stage and the kinds of things they might need to assess. Take feedback.

If they decide it is helpful, ask them to ensure that their annotations on the handout are clear, because this will be added to the school problem-solving policy.

Collect the annotated handouts and examples of children's work that demonstrate achievement of different aspects of problem solving at different levels.

Logic problems

This guidance is part of a series of materials from the Primary National Strategy designed to help all staff involved in the teaching of mathematics to:

- identify problems of a particular type and the strategies that children can use to solve them;
- construct teaching sequences for teaching problem solving;
- incorporate problem solving within the mathematics curriculum;
- develop children's reasoning and explanation skills;
- engage in whole-staff discussions on the above.

These materials are designed to follow on from the first CPD pack (DfES 0247-2004 G) which focused on 'Finding all possibilities' and Module 2, 'Finding rules and describing patterns'. It is recommended that those materials are used before these.

The materials together aim to support schools to:

- identify the principles of good practice when teaching problem solving;
- identify the progression in children's strategies that should inform learning and expectations.

The two previous units should have prompted schools to being deciding on their policy for improving the teaching and learning of problem solving. These materials are designed to help them to build on those decisions through a focus on logic problems.

Proposed model for use of these materials in school

This set of materials consists of:

- two suggested staff meetings, one of 65 minutes, the other of approximately 50 minutes;
- a set of lessons focusing on logic problems. Teachers are asked to use them with their own classes between the first and second staff meetings.

Primary National Strategy Ref: DfES TBA-2005

Module 3: Logic problems

Objectives

- To build on the work from the previous modules by broadening the understanding of problem solving
- To consider whole-school progression within problem solving

Notes to guide whole-staff discussion: meeting 1

65 minutes

Outline of staff meeting

1. Introduction, building on 'Finding all possibilities' and 'Finding rules and describing patterns'
2. Solving logic problems: a consideration of the features and strategies for solving logic problems
3. The problem-solving lessons: work in cross-phase groups to look at the main features and progression of the two types of logic problems
4. Preparing the lessons for teaching, including making decisions about focus and assessment
5. Where next?

Objectives

- To build on the work from the previous modules by broadening the understanding of problem solving
- To identify progression and strategies in solving logic problems
- To identify useful recording systems for logic problems
- To consider links with other areas of the mathematics curriculum
- To decide on an assessment focus

Key messages

- Children need to develop their skills in solving logic problems and puzzles.
- Children need to be given opportunities to practise explaining their mathematical thinking.
- Children need practice in identifying and prioritising the information that will help them solve problems.

Resources

- Copies of the school policy for problem solving, or notes on the principles agreed in previous staff discussions
- Handout 2 from the CPD 'Problem solving' pack (DfES 0247-2004G)
- 'Gold, silver or lead' – logic problem for staff meeting
- Copies of the lessons in the 'Logic problems' pack, two for each year
- If agreed at the last mathematical problem-solving meeting that it would be useful for planning and assessing, copies of the school's checklist on progression in problem solving
- Copy for each teacher of what you want them to give feedback on at the next staff meeting on problem solving

Introduction and solving a logic problem (20 minutes)

Ask teachers of children in the Foundation Stage and Years 1, 3 and 5 to sit together, and teachers of children in Years 2, 4 and 6 to sit together.

Note:
If there is more than one teacher from a given year in either group, you may want to split that group, but every resulting group needs at least one representative from all its years – all of Reception and Years 1, 3 and 5, or all of Years 2, 4 and 6.

Explain that we are continuing to look at the skills and strategies needed to help children become proficient problem solvers. Remind teachers that the Primary National Strategy has classified five different types of problem as follows:

- Finding all possibilities;
- Logic problems;
- Finding rules and describing patterns;
- Diagram problems and visual puzzles;
- Word problems.

Remind them that they have previously focused on the types of problem that involve finding all possibilities and finding rules and describing patterns, and that they taught two lessons between staff meetings and then discussed:

- the features of good problem-solving lessons;
- the principles of good practice when teaching problem solving;
- how they would help children to develop some of the important strategies and skills needed in problem solving in the school.

Remind staff what was agreed in previous meetings.

Say that in this meeting they are going to look at logic problems.

Solving a logic problem

Give out copies of Handout 2 from the CPD 'Problem solving' pack: Types of problem and appropriate strategies. Draw attention to the strategies identified under the heading 'Logic puzzles'. They are as follows.

- Identify the given facts and prioritise them.
- Look for any relationships and patterns in the information given.
- Use one piece of information at a time and see what effect it has, then keep one fixed and test the other.
- Choose and use a recording system to organise the given information.
- Check that the answer meets all the criteria.

One definition of logic problems is those problems where the relationship between criteria is used to reach a solution. When trying to solve logic problems it is particularly important to 'unpick' the language. Sometimes this language will be explicit, but sometimes it will be implicit.

Tell the staff that in order to understand the features of logic problems you are going to work through some together, considering which of the identified strategies they use.

Note:
The aim of this activity is to help staff understand the nature of logic problems, to examine some of the strategies used to solve them and to share approaches to solving them. Four problems of increasing difficulty are presented; it is not essential that all four problems are completed and judgement must be used about when it is appropriate to move on to the next section.

Problem 1

Give out the first of the 'Gold, silver or lead' problems. Tell staff that they can discuss the problem and make annotations on the handout and that they have to reach a consensus on the solution.

Give staff a minute before taking feedback.

Agree that the money is in the silver casket. Ask staff what strategies they used. These could include:
- using one piece of information at a time;
- crossing out which casket it couldn't be in as the information was revealed (a simple recording system);
- checking that the answer met all the criteria.

Problem 2

Give out the second problem. Explain that you will want the answer as well as a clear explanation of how teachers reached it, including the strategies they used. Give staff a couple of minutes to work through the problem and to agree their answer before taking feedback.

Ask each group which casket they decided held the money and to demonstrate how they reached this conclusion.

It is most likely that the groups used the strategies of:
- using one piece of information at a time, fixing that (as either true or false) and seeing what effect it had;
- ensuring that they met all the criteria as they tested each possibility, that is, that only one statement is false;
- devising a helpful recording system to help them to work logically. They might have used the following strategy to record their reasoning.

Gold	Silver	Lead	Outcome
true	true	false	silver
true	false	true	impossible
false	true	true	impossible

Where 'impossible' is written in the outcome column, it is because statements would contradict each other, making it impossible to have only one statement that was false (one of the criteria).

Agree that the answer is silver and that the label on the lead casket is false.

Problem 3

Give out the third problem. Tell them to take particular note of the language: 'No label contains more than one false statement.'

Say that again you want staff to be prepared to share their strategies, including their recording systems, with others to explain how they solved the problem. This can often be the most difficult thing when teaching children how to solve logic problems, so it is most important that they are able to explain to each other. A system for recording will help them to keep track of what they are doing while solving the problem, as well as help them communicate their reasoning.

Give staff about 4–5 minutes to work through the problem before taking feedback.

One possible strategy is as follows.

Step 1: Take the statements on each casket one at a time and identify the outcomes for true/true, true/false and false/true.

Gold	T–T: silver/lead – Wales	T–F: silver/lead – not Wales	F–T: gold – Wales
Silver	T–T: silver/lead – England	T–F: silver/lead – not England	F–T: gold – England
Lead	T–T: silver	T–F: gold	F–T: impossible, contradiction

Step 2: The statements on the lead casket only identify silver or gold, so we can exclude lead as a possible container.

Step 3: The only combinations of statements that identify gold contradict each other (Wales and England) so we can exclude gold. This leaves silver as the only container that can contain the money.

The combination could be gold T–T, silver T–F, lead T–T; or gold T–F, silver T–T, lead T–T.

Problem 4

Give out the fourth problem, again drawing attention to the language: 'on one label both statements were true, on one label there was one true and one false statement and on the other both statements were false.'

Say that again you will want them to explain how they got their answer. Give staff about 5–6 minutes to work through the problem before taking feedback.

One possible strategy is as follows.

Step 1: Test whether all the statements can be both true, both false, true then false, false then true:

Gold	T–T: possible	F–F: possible	T–F: possible	F–T: impossible, contradiction
Silver	T–T: possible	F–F: possible	T–F: possible	F–T: impossible, contradiction
Lead	T–T: possible	F–F: possible	T–F: possible	F–T: impossible, contradiction

Step 2: Draw a table of possible combinations.

Gold	T–T: silver	T–T: silver	T–F: lead	T–F: silver	F–F: gold	F–F: gold
Silver	F–F: gold	T–F: gold	T–T: lead	F–F: gold	T–T: lead	T–F: silver
Lead	T–F: silver	F–F: lead	F–F: lead	T–T: gold	T–F: silver	T–T: gold

There is only one combination where all three statements have the same outcome (lead). This also matches the criteria – one label has both statements true, another both false another true and false.

To summarise, draw out that:

- no calculations were involved in solving the problem, but the challenge became increasingly difficult because of the need to hold one or more pieces of information while interpreting other information;
- as the problem became more difficult there was an increasing need to use logic puzzle strategies:
  - prioritising given facts;
  - using one piece of information and seeing what effect it had, then fixing another;
  - developing a recording system;
  - very carefully checking that the answer met all the given criteria or information.

Remind staff that the skills and strategies they have identified will need to be taught to children.

Progression in solving logic problems (20 minutes)

Give staff copies of the logic problems materials (two per year group). Explain that, as before, it is expected that the whole school will use these materials to focus on this aspect of problem solving.

Remind staff of the layout of the lessons (they should be familiar with this from previous problem-solving materials).

- The objectives, vocabulary and necessary prior knowledge are identified at the beginning of each set of lessons. Staff will need to check that children have the necessary prerequisite knowledge, and teach what they do not have.
- Progression has been built into the lessons: the skills and strategies developed in the first lesson are built on in the second lesson.
- The lessons are set out with the 'main story' of the lesson in the middle column. Down the left-hand side there are suggestions on how the lesson might be modified. On the right-hand side there are illustrative thoughts behind the planning of the lessons and suggestions on aspects for feedback in the next staff meeting.
- The lessons are not to be used as a script, but adapted to different children's needs.

Tell staff that there is a difference between the problems for Years 2, 4 and 6 and those for Years 1, 3 and 5 (those for the Foundation Stage fit more with the latter group). The main differences are as follows.

- Those for Years 2, 4 and 6 involve number and children will need some number knowledge and skills, as identified on the first page of each activity.
- Those for the Foundation Stage and Years 1, 3 and 5 are not about number.
- Those for Years 2, 4 and 6 involve algebra: the use of the empty box to stand for different numbers; looking for relationships and patterns; making generalisations and applying those to new problems.

Say you want them to read carefully through the appropriate problems for the year groups they are in, identifying:

- the main features of the lessons;
- the progression through the years.

Notes:

In the Foundation Stage and Years 1, 3 and 5 these are likely to include the following.

- Each pair of lessons uses paired work and group work (varied in the Foundation Stage).
- There are good opportunities for Speaking and listening work.
- In every lesson children are asked to identify what the problem is asking them.
- Children are expected to justify their reasoning and decisions at every age.
- The importance of language is developed through the years. For example, children are asked to identify important words in the Foundation Stage and Year 1 (there is a focus on positional language in Year 1 and Year 3); in Year 3 'is not' clues are introduced (that is, negative as well as positive clues); in Year 5 this is developed to include 'cannot' and pupils need to pick up clues that are not explicit (see 'Nicknames').
- Children need to evaluate and prioritise information, identifying 'start' clues in Years 1, 3 and 5 and useful questions to narrow down options in the Foundation Stage;
- Children are encouraged to reflect on the impact of each clue on other information. In the Foundation Stage they reflect on the impact after each object is placed in a set; in Year 1 and Year 3 this impact is crucial to the positioning of objects. In Year 5 the impact of implicit clues allows some possibilities to be eliminated.
- In Year 3 children are encouraged to move objects around to solve a problem. They also need to think of a way of keeping track of information: 'Timetable' requires some recording in the form of crossing out possibilities. This is developed in Year 5 ('Nicknames') where systematic recording in the form of a table is necessary.
- Every problem involves checking the solution against given criteria. In the Foundation Stage children use one criterion at first and then more are introduced; this is developed through Year 3 (both problems) and Year 5 (particularly 'Tea for Two').

In Years 2, 4 and 6 these are likely to include the following.

- Each pair of lessons uses paired work and group work.
- There are good opportunities for Speaking and listening work.
- All problems involve number. Pupils need to be able to add and subtract numbers within 20 at Year 2, larger two-digit numbers at Year 4 and Year 6, and need to be able to double and halve numbers in Year 4.
- There is progression in an aspect of algebra through the years. In Year 2 different symbols are used to stand for unknown numbers; this is continued into Year 4. There is some use of this in Year 6 ('Albert Square').

- In Year 4 and Year 6 it is suggested that pupils devise their own similar problems to extend the activity. By doing this they will be applying their understanding of the use of different symbols to stand for unknown numbers.
- Prioritising information and identifying the starting point is very important in every year. In Year 6 there is redundant information too.
- From Year 2 to Year 6 pupils see the impact that solving one part of the problem has on another part. Because of this they need to check the information given at each stage, but in Year 6 they need to keep a lot of information in their heads while they are working through the problems, not just at the end of each stage.
- Recording is important in each year. In Year 2 and Year 4 pupils are given a table to complete; they need to realise that this form of recording can often be a help in solving problems because it makes the information clearer and so easier to use.
- In Year 4 children are asked to generalise about strategies; this is continued in Year 6.

Considering the implications for teaching (10 minutes)

Either take whole-staff feedback considering:

- the main features and progression each group has identified;
- the common features identified;
- the implications for teaching, particularly what the pupils in each year need to be secure about in order to access the problems of the following year. (The necessary prior knowledge identified in the lessons plans will be helpful, but encourage teachers to talk about other things they might have identified.)

Or:

- ask teachers to do this in cross-year groups, for example Foundation Stage, Year 1 and Year 2; Year 3 and Year 4; Year 5 and Year 6.

Notes:

The implications are likely to include the following.

- Children need to be given lots of opportunities to explain their reasoning from the Foundation Stage through to Year 6 in pairs, groups and to the whole class.
- Children need to identify key words in problems and to articulate what a problem is asking.
- Prioritising information, identifying starting points and reflecting on the impact of clues have implications for multi-step problem solving.
- Logic problems can be brought into many areas of the mathematics curriculum, for example number, shape and space and handling data.
- Children need to become secure with reading information from tables and drawing their own tables of data as they progress through the school, because it is valuable for clarifying information and solving a range of problems in a variety of subjects.
- Children must have key numerical knowledge and skills by the end of each year.
- Teachers and children need to be aware of the link between solving 'empty box' problems and algebra so that, for example, Year 6 pupils are able to use a symbol to stand for an unknown number when they are trying to solve a problem. For example, in the 'Albert Square' problem, information can be represented as 15 + 15 + 36 = + .
- Checking solutions against criteria (or to see that it makes sense) is a skill that can be developed in all areas of problem solving, not just logic problems.

Lesson preparation and the assessment checklist (10 minutes)

Remind teachers that they are going to teach the two logic problem lessons with their classes before the next staff meeting.

Say that they will be familiar with the content of the lessons from the previous activities, but you are now giving them a few minutes to consider:

- the principles for planning and teaching problem-solving lessons that were agreed in an earlier staff meeting;
- the problem-solving assessment focus.

Remind them of the principles that were agreed.

If the following points do not form part of the principles you might want staff to consider them.

- How will you introduce the problem to your class and ensure that they understand it?
- What resources do you need so that all the children can access the problem at their level?
- How can we remove any barriers that might prevent children from being successful?
- How far will you expect children in your year to get with the given problem? (Consider the ability range.) What strategies might you expect them to use? Are there strategies you will want to develop?
- What recording will you expect from children?
- What kinds of explanation will you expect?
- What skills do children need to access the problem? Do they have these? When will you teach them?

If there was agreement about its use at the last problem-solving meeting, offer the following suggestions on the checklist on progression in problem solving.

- While preparing lessons ask teachers to identify those statements they would expect to be focusing on. You might want to collect these at the end of the meeting so that you have an overview of what each school year is expecting to focus on.
- Decide on any statements teachers might want to focus on in assessment. They might want to consider this after teaching the first lesson and use the second lesson for some assessment.
- Consider how the assessment might be carried out, for example teacher observing with a group; teaching assistant observing a group; assessing the work produced by the class or a group; or asking a group to explain their work to the class.

Where next? (5 minutes)

Agree with the staff when they might teach the two lessons with their classes and when the next staff meeting will be to discuss children's progress.

If appropriate, agree on the use of the checklist.

Say that at the next staff meeting you would like them to bring examples of children's work from their classes that show:

- that they have achieved the lesson objectives;
- the use of some of the strategies for solving logic problems identified in Handout 2, particularly:
  - identifying facts and prioritising them;
  - checking that their solutions meet the given criteria;

- a variety of methods of recording, where appropriate, that helped children keep track of their reasoning;
- examples of children's explanations (written, or notes taken from oral explanation). Remind staff that children cannot always explain their thoughts succinctly and it is therefore necessary to give them opportunities to explain orally.

Note:
It might be useful to give staff a copy of this list, adjusted as desired, to remind them.

Notes to guide whole-staff discussion: meeting 2

50 minutes

Outline of staff meeting

1. Introduction; reminder of focus
2. Feedback on lessons
3. Review of the checklist on progression
4. Where next?

Objectives

- To consider children's use of strategies when solving logic problems;
- To review decisions made about the whole-school approach to problem solving in the light of the focus on logic problems
- To review the use of the problem-solving checklist as an aid for planning and assessment
- To decide the timing of a review of progress in problem solving

Resources

- Copies of the lessons in the 'Logic problems' pack
- Examples from teachers of children's work on logic problems
- Copies of teachers' annotated checklists on progression, if used
- Copies of the school policy for problem solving or notes on the principles agreed
- Optionally, a list of points to consider as teachers give feedback to colleagues about the lessons

Introduction (2 minutes)

Remind teachers that you asked them to bring to the staff meeting examples of children's work from their classes that show:

- that they have achieved the lesson objectives;
- the use of some of the strategies for solving logic problems identified in Handout 2, particularly:
  - identifying facts and prioritising them;
  - checking that their solutions meet the given criteria;
- a variety of methods of recording, where appropriate, that helped children keep track of their reasoning;
- examples of the children's explanations (written, or notes taken from oral explanation).

Feedback on lessons (25–30 minutes)

Ask teachers to share the examples they have brought. Teachers, particularly those of younger children, will also have notes based on their observations.

Ask teachers to consider the following points when their colleagues are giving feedback, making notes if appropriate.

Notes:
You might want to consider asking different staff to make notes on different points, or give staff a list of the points to focus on.

- Is there progression in the child's ability to use logic-problem strategies through the school, particularly including identifying and prioritising facts and checking solutions?
- Did children recognise strategies they might have learned in lesson 1 that could be applied in lesson 2?
- What recording strategies did children have? Were they using strategies that they had learned in other aspects of problem solving, for example working systematically or making an ordered list or table?
- How good are children at explaining their reasoning when working with others and when asked to summarise their work? Is there anything we need to do to improve this?
- Which combinations of working were the most helpful for children to develop reasoning, for example working in pairs, or mixed-ability groups?
- Were there any aspects that teachers found particularly difficult to teach? How did they or could they get around these?
- Were there any aspects that children found particularly difficult? How were these overcome, or how could they be?
- Was there a difference between children's responses to the numerical problems (Years 2, 4, 6) and the non-numerical problems (Foundation Stage, Years 1, 3, 5)? If so, are there any implications for teaching?

Discuss any issues that arise as a result of the feedback, particularly those that have implications for the whole school.

Use of checklist on progression (10 minutes)

Where staff used the checklist on progression as a guide to deciding the focus of the lessons and for assessment ask for feedback on how useful it was.

You might want to consider the following.

- Which statements were used as a focus for the lesson or for assessment, and by which years? If statements were used by a wide range of years, were teachers able to make a distinction between expectations for different abilities or year groups? How was this done?
- Is the bank of statements helpful as a guide for planning and assessment?
- Are there statements that need modifying? Are there statements that should be added?

Where next? (10 minutes)

As a result of both the feedback and discussions decide on the next steps for your school. These are likely to include:

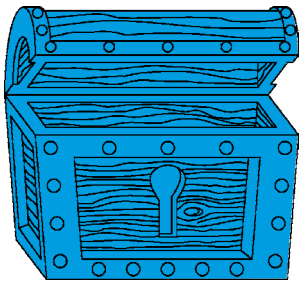
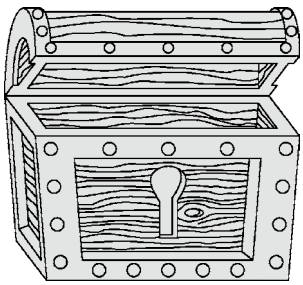
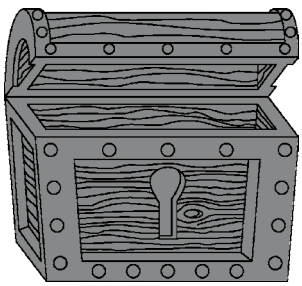
- how the outcomes of the focus on logic problems can be integrated into decisions already made about the whole-school approach to problem solving;
- whether logic problems have given staff any more ideas about integrating problem solving into the mathematics curriculum. If they have, how will this be done?
- whether there are aspects of children's reasoning and explanation skills that need to be developed and how this can be done through other areas of the curriculum, for example by considering the Speaking and listening policy;
- how useful the checklist on progression is for staff in helping plan and assess different aspects of problem solving. If it is useful, how will it be used?
- how staff will support each other in developing the teaching of problem solving, for example joint planning, joint teaching, peer observations, or developing a 'bank' of different types of problem;
- the continuing development of a portfolio of children's work to show expectations in problem solving for each year;
- arranging a staff meeting to review the school's approach to problem solving in two terms. At this meeting there is likely to be a review of:
  - progress in problem solving evident in children's approaches and work;
  - the assessment of children's abilities in problem solving.

Problems for staff meetings

Problem 1

Aysha is deciding which of her children to leave her wealth to. She decides that she will leave it to the most logical, and so she sets them some challenges.

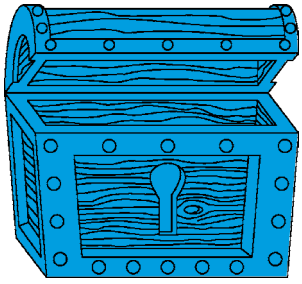
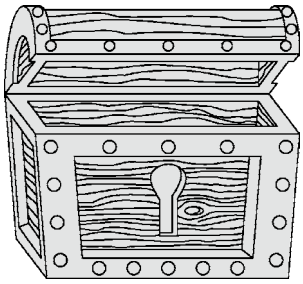
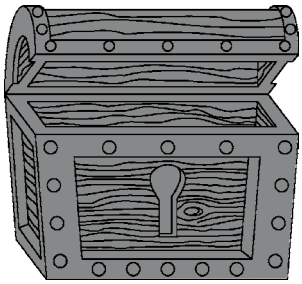
There are three caskets, each with a label. The money is in just one of the caskets. Which casket is it in if all the statements are true?

Gold	Silver	Lead
		
The wealth is not in here	The wealth is not in the lead casket	The wealth is in either the silver or the gold casket

Problem 2

Too many of Aysha's children identified the correct answer – silver – so she devised a second test.

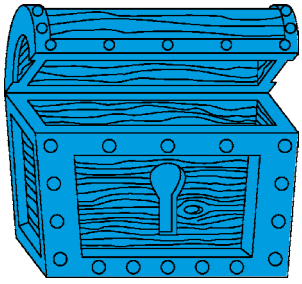
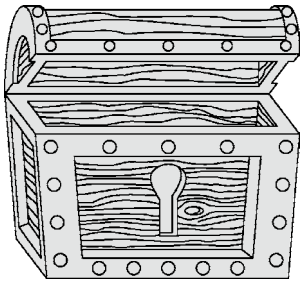
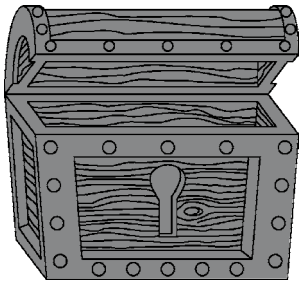
In this case one and only one statement is false.

Gold	Silver	Lead
		
The wealth is not in here	The wealth is not in the gold casket	The wealth is in here

Problem 3

Gemima inherited her mother's money and she had to devise tests for her own children when the time came.

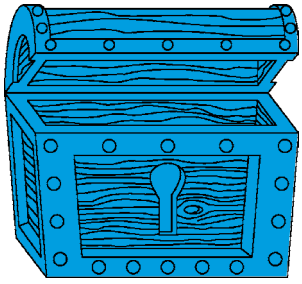
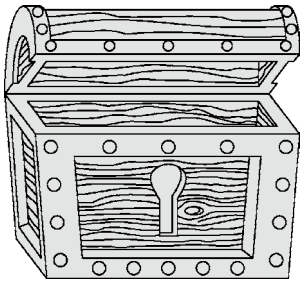
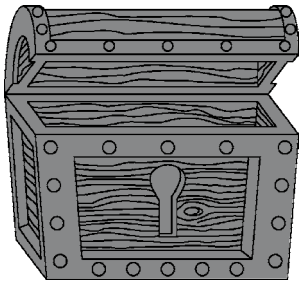
No label contains more than one false statement.

Gold	Silver	Lead
		
1. The wealth is not in here 2. The money was minted in Wales	1. The wealth is not in the gold casket 2. The money was minted in England	1. The wealth is not in here 2. The wealth is really in the silver casket

Problem 4

Gemima's children were also clever, so she devised another test.

In this test, on one label both statements were true, on one label there was one true and one false statement, and on the other both statements were false.

Gold	Silver	Lead
		
1. The wealth is not in here 2. It is in the silver casket	1. The wealth is not in the gold casket 2. It is in the lead casket	1. The wealth is not in here 2. It is in the gold casket

Handout

Making and monitoring decisions to solve problems	Year Group
Can use given equipment appropriately to help to solve problems.	
Can choose the equipment needed and decide how to use it to solve problems.	
Can decide what would be the best equipment to use and if it's not available to find something else which will help.	
Can use mathematics in everyday situations in the classroom (e.g. can measure things out when cooking. Knows to count and compare if teacher asks, 'how many girls are away today?').	
Can identify key words and facts needed to solve a problem and carry out the appropriate processes	
Recognises when there is more information than needed to solve a problem.	
Can check methods and adjust to correct work.	
Can check a solution meets given criteria	
Can find their *own way* of solving a problem.	
Can decide when a table or list is a useful device.	
Sometimes has more than one way of finding a solution.	
Can compare different methods and solutions and decide which is more efficient.	
Recognises how a method can be applied to solve similar problems	
Can simplify the problem by trying easier cases.	
When having difficulty with a problem can stop, think about it and try a different approach.	
Can break a complicated task up into smaller simpler steps to help make a start.	
Can carry out separate steps in a problem and combine them to get the solution.	
Can find lots of possibilities	
Has a system for finding the possibilities, e.g. starts with the smallest number	
Can make a list of possibilities working systematically.	
Checks for repeats when listing all possibilities.	
Can decide if all possibilities have been found.	
Can use one piece of information given in a problem and work systematically to see what effect it has.	

Handout

Developing mathematical language and communication	Year Group
Knows some mathematical words and uses them when talking about work.	
Can explain a pattern.	
Can explain what they have done by talking about it using correct mathematical language, symbols, pictures or diagrams to help.	
Can begin to use the language of reasoning eg 'It can't be… because…' 'It could be… because…'	
Develops the language of reasoning eg 'If I do this… then this will have.... effect.'	
Can explain to someone what was done and why it worked.	
Can restate the problem in their own words, asking appropriate questions to clarify a problem.	
Can read and interpret simple diagrams and instructions.	
Can represent a problem with a diagram where appropriate	
Can look at some different methods and describe their features.	
Can choose their *own* way of recording what has been done.	
Can explain why they chose to present work in a certain way.	
Make jottings to help solve a problem.	
Annotate diagrams or pictures to simplify the problem.	
Keep track of progress by recording the key steps/calculations.	
Can interpret what they record to find any mistakes.	
Can explain in writing what has been done using mathematical vocabulary, symbols, pictures or diagrams.	
Can compare ways of presenting something and say which is better and why	

Handout

Developing mathematical reasoning	Year Group
Can sort objects etc to meet a given criterion	
Can sort objects etc to meet two or more given criteria	
Can choose own one or more criteria for sorting and apply accurately	
Can follow a simple pattern and extend it, e.g. ❍, ☐, ❍, ☐,..... or hop, skip, clap, hop, skip, clap,.....	
Can recreate and describe a simple pattern (number, shape, object etc).	
Can predict what comes next in a sequence and explain why.	
Can explain a pattern.	
Can apply a rule to predict whether a number, shape or object will be in sequence or not	
Tries to search for patterns or reasons why things work out as they do e.g. How can you get from one square number to the next, is there a pattern?	
Can explain a pattern through a general statement	
Can create an algebraic formula for the pattern	
When solving a problem, can identify and prioritise given facts	
When solving a problem can explain a starting point and the following steps	
Can check a solution meets given criteria	
Is able to respond to 'What if…', 'What would you try next?' questions	
Can respond to questions such as 'How do you know that?' 'Why do you think that?'	
Can try out *if*……. *then* type reasoning	
Can ask their own appropriate what if…? questions of themselves or others.	
Can investigate a mathematical statement to determine whether it is true, false or sometimes true e.g. 'two odd numbers always add up to make an even number'; 'to multiply by 10 you add a nought'	
Can look at what's been found and make a general statement about it (e.g. 'taller people tend to have bigger feet but boys have bigger feet, on average, than girls')	
Can use mathematical language to create a general statement orally	
Can use mathematical language to create a general statement in writing.	