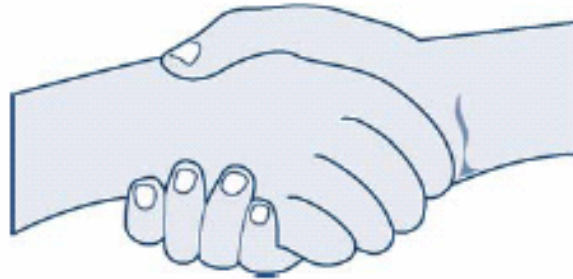


## Handshakes



- Everyone in this room shakes hands with everyone else.  
How many handshakes are there?

### Simplifications

- How many handshakes would there be for 3 people? And 5 people?

### Challenges

- How many handshakes would there be for 100 people?
- Generalise using words or symbols.

### Appropriate strategies for Finding Rules and Describing Patterns

- Decide on the information you need to describe and continue the pattern
- Give examples to match a given statement and ones which do not
- Describe a rule of a pattern or relationship in words or pictures
- Predict the next few terms in a sequence to test a rule
- Use a rule to decide whether a given number will be in a sequence or not

## Handshakes Problem

No. of people	1	2	3	4	5	6
No. of handshakes	0	1	3	6	10	15
Pattern	0	1+0	2+1+0	3+2+1+0	4+3+2+1+0	5+4+3+2+1+0
Formulae	For an even number (n) of people: handshakes= $(\frac{1}{2} n) \times (n-1)$					
	For an odd number (n) of people: handshakes= $\frac{1}{2} (n-1) \times n$					

For a group of 100 people:

$$\begin{aligned}\text{Number of handshakes} &= (\frac{1}{2} n) \times (n-1) \\ &= 50 \times 99 = 4950\end{aligned}$$

For a group of 75 people:

$$\begin{aligned}\text{Number of handshakes} &= \frac{1}{2} (n-1) \times n \\ &= \frac{1}{2} (74) \times 75 = 2775\end{aligned}$$

### Linked Problems:

- Five families on holiday, all send a postcard to every other family.
- A group of people play a badminton tournament; each person must play every other person once.
- Problem can be done visually - a circle with 5 points equally spaced around the circumference; if each point is joined to every other point with a straight line, how many lines will be drawn? Try other numbers of points - what patterns are created?(this problem is sometimes known as Mystic Rose)
- Problem can be shown practically - stand children in a circle, join one child to every other child with pieces of string, move on to second child etc. This provides a record of the handshakes.
- At a party, half of the guests are girls and half are boys. If the girls only say 'hello' to other girls, and the boys to other boys - how many 'hellos' will there be?

## Handshakes Problem

### Level guidance

#### Level 1:

- Pupils able to 'act out' problem correctly within a small group
- Pupils able to count accurately the number of handshakes
- Simple recording of findings in pictures or diagrams (adult support allowed)
- Able to understand the idea (through talk) that the number of handshakes increases with the number of people

#### Level 2:

- Pupils work practically in groups to 'act out' problem, with some independence
- Record results using simple diagrams (stick figures, crosses to represent people) and mathematical symbols (link correct digit to matching diagram)
- Begin to order results (eg. in size of group) and check for repeats - after adult suggestion if necessary
- Able to explain their methods (orally or written) of solving the problem
- Able to predict what would happen to the pattern if the group was larger, with reference to results already found (0,1,3,6,10,... prediction that next number will be in early teens acceptable) giving simple reasons

#### Level 3:

- Pupils move from 'acting out', to working with practical apparatus or diagrams to solve the problem
- Begin to tackle the problem systematically (1 person, 2 people, 3 people, ...)
- Use appropriate diagrams and symbols to represent their work - enabling their results to be checked
- Able to describe a simple rule found in the results (0,1,3,6,10,... add 1, add 2, add 3, ...)
- Able to predict and check the next few terms in the sequence to test their rule
- Able to explain their methods (orally or writing) and use simple mathematical reasoning linked to their prediction

#### Level 4:

- Aware that problem should be tackled in a systematic way, (initial attempts may not be systematic, but approach should change once problem is understood)
- Results recorded in an organised and clear way (ordered list or table)
- Aware that results may lead to a pattern, predict and check the next few terms to test the pattern's rule
- Be able to describe pattern's rule in words, use mathematical reasoning to find further results (eg. a group of 20 people)

#### Level 5:

- Independently decide to work systematically in finding and checking results
  - Communicate findings in an appropriate way, with sufficient detail to enable others to interpret their work
  - Able to describe more complex patterns (0, 1, 1+2, 1+2+3, 1+2+3+4,...) and recognise familiar groups of numbers (triangular numbers)
  - Able to explain, using mathematical reasoning, the link between the problem and the pattern discovered (linking the descending addition pattern to the fact that two people are needed to shake hands which counts as one handshake that does not need to be repeated)
- ❖ (able to explain a general relationship in words, and then as a formula using letters as symbols - *not necessarily appropriate in this problem as formula is complex*)